Isometric Projection

An isometric projection of a cube aligns two opposite corners so that the projection appears as a hexagon composed of six equilateral triangles.

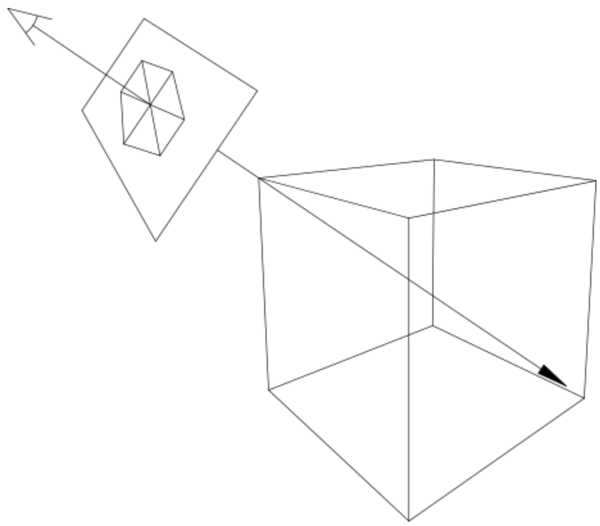

Fig 1

Perspective is not a part of an isometric projection and the x and y coordinates are derived from the corresponding distances of a point in three dimensional space from two perpendicular planes with no convergence on the view point.

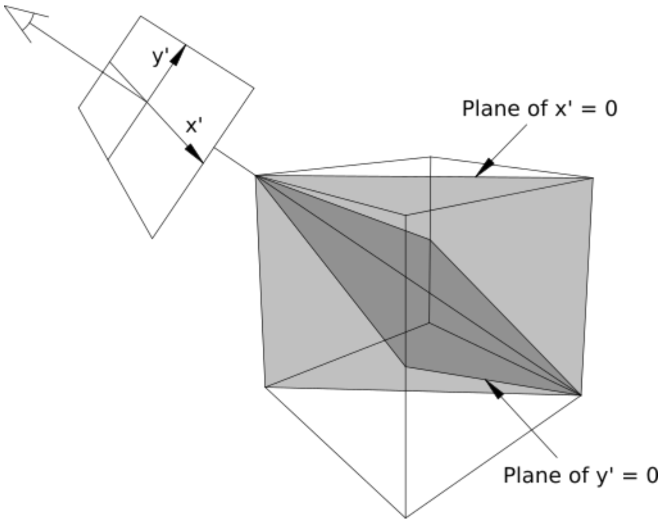

Fig 2

The shape of an isometric projection remains the same regardless of it's displacement relative to the origin so we may pick any convenient point as the origin and perform two dimensional translation afterwards if necessary.

X Coordinate

The x coordinate is obtained by calculating the distance of a point from the plane of x' = 0.

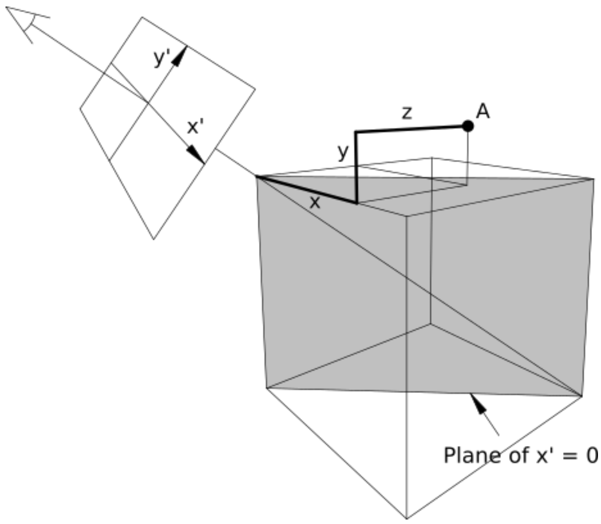

Fig 3

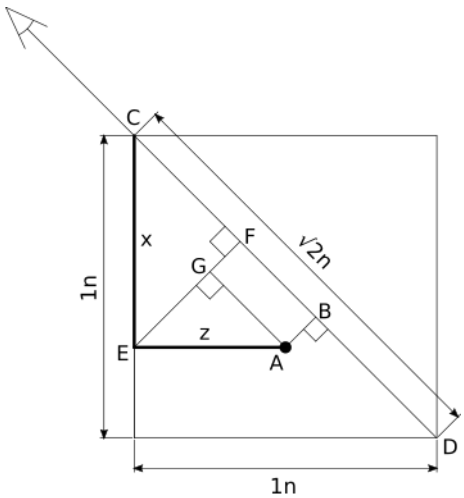

Fig 4

In Fig 4 the plane of x' = 0 is represented by the line CD and the distance of point A to the plane is the line BA. AEG and ECF are similar triangles with sides in the ratio √2:1:1.

$$BA = FE - GE$$

$$FE = x / \sqrt{2}$$

$$GE = z / \sqrt{2}$$

$$CB = CF + GA$$

$$CF = x / \sqrt{2}$$

$$GA = z / \sqrt{2}$$

$$CB = (x + z) / \sqrt{2}$$

$$x' = (x - z) / \sqrt{2}$$

Y Coordinate

The x coordinate is obtained by calculating the distance of a point from the plane of y' = 0.

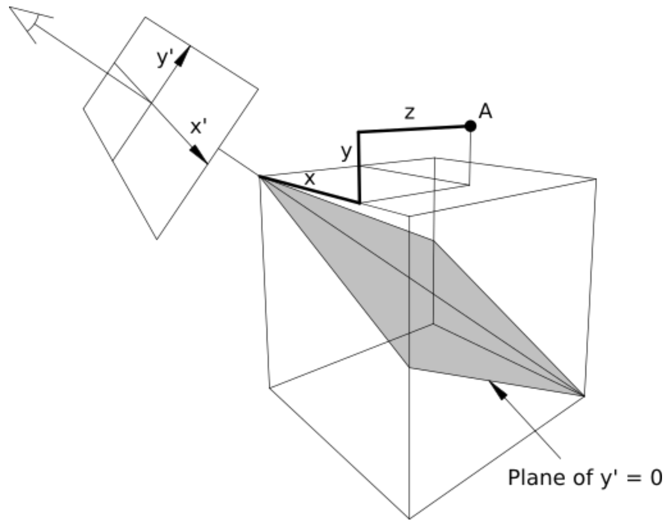

Fig 5

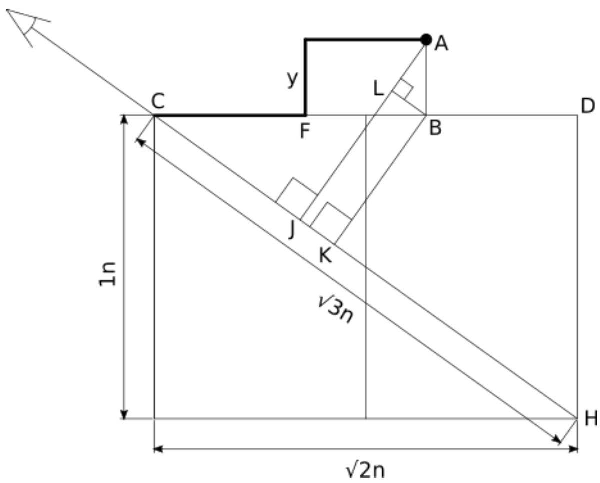

Fig 6

In Fig 6 the plane of y' = 0 is represented by the line CH and the distance of point A to the plane is the line JA. BAL and BCK are similar triangles with sides in the ratio √3:√2:1. Fig 6 is orthogonal to Fig 4 and the points C, B, D and F correspond in the two diagrams such that CB in Fig 4 equals CB in Fig 6.

$$JA = LA + KB$$

$$LA = \sqrt{2} \cdot y / \sqrt{3}$$

$$KB = CB / \sqrt{3}$$

$$= (x + z) / (\sqrt{2} \cdot \sqrt{3})$$

$$y' = \sqrt{2} \cdot y / \sqrt{3} + (x + z) / (\sqrt{2} \cdot \sqrt{3})$$

$$= (x + 2y + z) / (\sqrt{2} \cdot \sqrt{3})$$

QCAD

QCAD has tools for generating isometric projections from plan, side and front elevations but not from two dimensional drawings of elements that are not parallel or perpendicular to these standard orientations. There is however a tool to apply a generic transformation based on a two dimensional matrix.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = ax + by$$

$$y' = cx + dy$$

For a 2 dimensional elevation let z = 0

$$x' = (x - 0) / \sqrt{2}$$

$$= x / \sqrt{2}$$

$$y' = (x + 2y + 0) / (\sqrt{2} . \sqrt{3})$$

$$= (x + 2y) / (\sqrt{2} . \sqrt{3})$$

$$= x / (\sqrt{2} . \sqrt{3}) + y . (\sqrt{2} / \sqrt{3})$$

Therefore the matrix elements for a isometric projection are :

$$a = 1 / \sqrt{2}$$

$$b = 0$$

$$c = 1 / (\sqrt{2} . \sqrt{3})$$

$$d = \sqrt{2} / \sqrt{3}$$

Arbitrary Rotation and Inclination

Fig 7 shows a two dimensional plane with points (w,h) that has been rotated θ degrees and tilted γ degrees with respect to a three dimensional space with points (x,y,z).

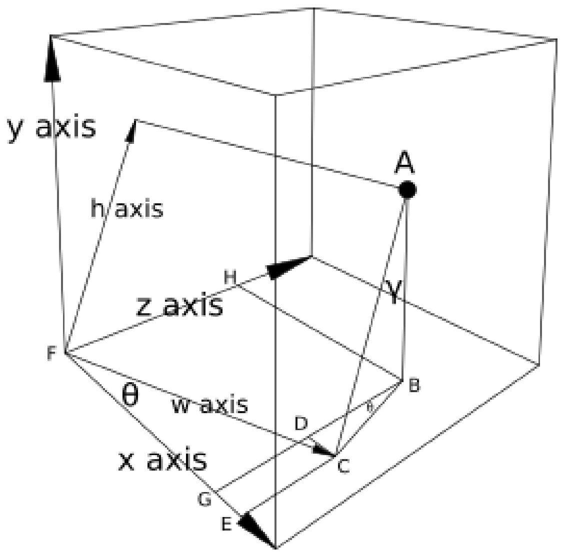

Fig 7

Given w, h, θ and γ it is possible to calculate the x y and z coordinates.

$$
\begin{aligned}
x &= FG \\
y &= BA \\
z &= GB \\
FG &= FE - DC \\
GB &= EC + DB \\
BA &= h.\cos(\gamma) \\
FE &= w.\cos(\theta) \\
DC &= BC.\sin(\theta) \\
EC &= w.\sin(\theta) \\
DB &= BC.\cos(\theta) \\
BC &= h.\sin(\gamma) \\
x &= w.\cos(\theta) - h.\sin(\gamma).\sin(\theta) \\
y &= h.\cos(\gamma) \\
z &= w.\sin(\theta) + h.\sin(\gamma).\cos(\theta)
\end{aligned}
$$

These can then be converted into an isometric projection using the X and Y coordinate formula above.

$$x' = (x - z) / \sqrt{2}$$

$$y' = (x + 2y + z) / (\sqrt{2} . \sqrt{3})$$

$$x' = (w.\cos(\theta) - h.\sin(\gamma).\sin(\theta) - (w.\sin(\theta) + h.\sin(\gamma).\cos(\theta))) / \sqrt{2}$$

$$= w.(\cos(\theta) - \sin(\theta)) / \sqrt{2} - h.\sin(\gamma)(\cos(\theta) + \sin(\theta)) / \sqrt{2}$$

$$y' = (w.\cos(\theta) - h.\sin(\gamma).\sin(\theta) + 2.h.\cos(\gamma) + w.\sin(\theta) + h.\sin(\gamma).\cos(\theta)) / (\sqrt{2} . \sqrt{3})$$

$$= w.(\cos(\theta) + \sin(\theta))/ (\sqrt{2} . \sqrt{3}) +$$

$$h.(\sin(\gamma).\cos(\theta) + 2.\cos(\gamma) - \sin(\gamma).\sin(\theta)) / (\sqrt{2} . \sqrt{3})$$

Therefore the matrix elements are :

$$a = (\cos(\theta) - \sin(\theta)) / \sqrt{2}$$

$$b = - \sin(\gamma)(\cos(\theta) + \sin(\theta)) / \sqrt{2}$$

$$c = (\cos(\theta) + \sin(\theta))/ (\sqrt{2} . \sqrt{3})$$

$$d = (\sin(\gamma).\cos(\theta) + 2.\cos(\gamma) - \sin(\gamma).\sin(\theta)) / (\sqrt{2} . \sqrt{3})$$