

## On the topic on the QCAD forum about curve fitting while exploding splines.

<https://www.qcad.org/rsforum/viewtopic.php?p=21530>

Started with setting the startpoints of the curves at about the same place.

The eMachineShop curve overlapped at start- and endpoint x589.. y-4995.. for 0.005367 units.

Corrected this and relocated the startpoint.

The spline overlapped at the endpoint x1701 y-5530 for a full segment.

Knotpoint with index 909 of 913 is already back at the startpoint as is knotpoint 912 again.

Not corrected.

Added layers, moved text to 'Legenda'

In the adapted file:

Selecting the spline and hovering over the reference points highlighted with an active function eg 'Draw Single Point' or PO.

We observe that from the start every 3th controlpoint is called an intersection and is met by the two QCADs curves.

Spline segments are between two such knotpoints.

The two adjacent references are endpoints of the governing vectors at that knotpoint.

Every spline segment is divided in an even number of QCAD polyline arc segments by factor 2,4,8... $2^n$ .

Exceptions are probably found where two resulting polyline arc segments are coradial and simplified.

>> Seems Biarc fitting and splitting up the segments in two until the approximation criteria is met.

>> Andrew mentioned this is biarc fitting in a later post (Tue Aug 21, 2018 6:48 pm)

QCAD polyline arc segments that make up one spline segment are all in perfect tangency to each other.

> Angle property at vertex = 180 degrees.

But even for rather smooth parts of the spline curve this is not necessarily so at the knot points.

> Angle property at vertex is not necessarily 180 degrees.

Two reasons:

- Two adjacent vectors of a knot are not necessarily at 180 degrees.

(Firts larger deviation @ Spline/39 174.877148°; Qcad 0.50/28 174.877148°; Qcad 0.05/56 174.877148°; eMach/28 171.31904°)

- The QCAD arc segments are tangent to the vectors at the knots with an accuracy of some hundred thousands of degrees.

(Probably due to mathematical accuracy limits opposed by the calculations with the extra large and/or extra small numbers involved)

Biarc-fitting main constraints are tangency to begin and end vector and tangency between the two arcs.

These constraints will solve to not one but a field of solutions.

Reading up about methods there is often mentioned the use of unified vectors but mostly later also about a ratio of vector weights.

This ratio will also define the solution field.

In most cases the vector length here differs with the segment but are fairly in the ratio of 1/1 per segment.

Already around the 4th spline knotpoint (start = 1) I would like to investigate the QCAD 0.50 results.

The two short and the two very flat bends troubled me.

How do these compare to other possible solutions, if any?

As I have read there can be none, one, two or a field of solutions.

Tangency to begin and end vector let us draw a line where the center of any solution arc would be.

This is a line orthogonal to the vector and through the knotpoint.

The direction this line is cast depends on how the spline is 'bulging' at a distance of the knotpoint.

When this centers lines are opposed the distance between the two centers is the sum of the radii.

When at the same side a different approach can be used.

Both can be solved mathematically to a multitude of pairs of solutions.

Which pair that is used depends on other constraints.

In this case the next important constraint should be optimal fitting.

For: Spline 1th segment; Polyline QCAD 0.50 segment 1&2.

Given the nature of the spline segment two arcs with similar radii were chosen.

The magnitude of the radii tend to converge to 65 units. Solved to RA = 65.00, RB = 63,21...

The solution is similar to the QCAD 0.50 polyline arcs.

The local fitting error is in the order of 0.25 units.

For: Spline 3th segment; Polyline QCAD 0.50 segment 6&7.

It appears that the biarc fitting for the QCAD 0.50 polyline is done with the radii in the same direction.

Less obvious but RA should point up and RB down.

Solved to RA = 100.00, RB = 141.97... (!! vs QCAD RA = 4969.21..., RB = 18.46...)

Fitting error 0.12 (versus QCAD 0.50 polyline 0.17).

And then realized the first arc should be flatter.

Solved to RA = 300.00, RB = 117.35...

The fitting error drops down to 0.04 units, or less than 1/4 of that of the QCAD solution with the same amount of arcs per segment.

For: Spline 4th segment; Polyline QCAD 0.50 segment 8&9.

Same remarks as for segment 6&7.

RA should point down and RB up.

Solved and iterated to locate the 'sweetspot' in 6 solutions.

Fitting error will be in the order of 0.05 units in between solution 5 and 6.

Only 17% of the error of the QCAD solution.

As this continues the spline in question could probably be optimal represented by:

$(303\text{knots}-1\text{double}) + 1\text{forsegmets} \times 2 = 606 \text{ arcs ...}$

give or take 800 nodes with a fitting error in the order of 0.10 units.

Conclusion:

+ QCAD explodes splines with the given approximation criteria met.

? What accuracy does QCAD use when it auto explodes? That of the AppPrefs or a fixed value?

+ QCAD returns Biarcs that connect in perfect tangency for the same spline segment.

+ On the other hand QCAD preserves the tangent angle at the spline knotpoint.

- QCAD may cast arcs for Biarc-fitting in an odd way.

? What is the QCAD approach and how is this coded?

Lots of quadratic equations opens lots of choices and are prone to errors.

- It returns Biarc solutions as soon as the given approximation criteria is met.

-- QCAD does not find the best solution in a Biarc-fitting solution field.

Regards.

CVH